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[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1136 **D**

Unique Paper Code : 2352571101

Name of the Paper : DSC: Topics in Calculus

Name of the Course : **B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **Two** parts from each question.

3. **All** questions carry equal marks.

1. (a) If $f(x)=\begin{cases} x\dfrac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}, & x\neq 0 \\ 0 & x=0 \end{cases}$. Show that f is continuous but not differentiable at $x=0$.

P.T.O.

(b) State Leibnitz theorem for finding the n^{th} differential coefficient of the product of two functions.

If $y = (\sin^{-1}x)^2$, prove that

$(1 - x^2)xy_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0.$

(c) State Euler's theorem on homogeneous functions.

If $u = \tan^{-1}\frac{(x^3 + y^3)}{(x - y)}$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

and $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 4u - \sin 2u$.

2. (a) Show that the function f defined as $f(x) = |x - 1| + |x + 1| \ \forall \ x \in R$ is not derivable at the points $x = -1$ and $x = 1$ and is derivable at every other point.

(b) If $y = e^{m \sin^{-1}x}$, show that

$(1 - x^2)xy_{n+2} - (2n + 1)y_{n+1} - (n^2 + m^2)y_n = 0$ and hence find $y_n(0)$.

(c) If $u = \sin^{-1}\frac{(x + y)}{(\sqrt{x} + \sqrt{y})}$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u.$$

3. (a) State Cauchy's mean value theorem. Verify the Cauchy's mean value theorem for (i) $f(x) = e^x$, $g(x) = e^{-x}$ in [0,1] and (ii) $f(x) = \sin x$, $g(x) = \cos x$ in $\left[-\frac{\pi}{2}, 0\right]$.

(b) State Taylor's theorem with Lagrange's form of remainder. Show that sin x lies between $x - \frac{x^3}{6}$ and $x - \frac{x^3}{6} + \frac{x^5}{120}$ $\forall x \in R$.

(c) Determine the values of p and q for which

$$\lim_{x \to 0} \frac{x(1 + p\cos x) - q\sin x}{x^3}$$ exists and equals 1.

4. (a) State Taylor's theorem with Cauchy's form of remainder. Use Taylor's theorem to prove that

$$1 + \frac{x}{2} - \frac{x^3}{8} < \sqrt{1+x} < 1 + \frac{x}{2} \text{ if } x > 0.$$

(b) State Lagrange's mean value theorem. Verify Lagrange's mean value theorem for the function $f(x) = \sqrt{x^2 - 4}$ in the interval [2, 4].

P.T.O.

(c) Evaluate $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$.

5. (a) Find the asymptote of the curve

$$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0.$$

(b) Trace the curve $y(x^2 + a^2) = a^3$.

(c) Find the reduction formula for $\int \cos^n x \, dx$, $n \geq 2$.

Hence find $\int \cos^4 x \, dx$.

6. (a) Determine the position and nature of double points on the curve

$$x^4 - 2y^3 - 3y^2 - 2x^2 + 1 = 0.$$

(b) Trace the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$.

(c) Prove that $\int_0^{\pi/2} \sin^{2n} x \, dx = \frac{2n!}{2^n (n!)^2} \cdot \frac{\pi}{2}$.